NUMERICAL SIMULATION OF NONEQUILIBRIUM PROCESSES IN AN ELECTRON–HOLE PLASMA IN BINARY HETEROSTRUCTURES.

2. COMPUTATIONAL EXPERIMENT

V. A. Nikolaeva, V. I. Ryzhii, and B. N. Chetverushkin

UDC 539.219.1:533.9

The results of a computational experiment simulating a dense electron–hole plasma in binary heterostructures are presented.

Heating of the electron–hole plasma (EHP) in semiconductor structures can substantially affect their properties. Such heating is linked to the fact that in energy of the carriers (electrons and holes) injected through the heterojunction substantially exceeds the thermal energy. Naturally, an appreciable deviation of the average energy of electrons and holes from the thermal energy occurs if the energy transfer from the EHP to the lattice is characterized by quite long energy relaxation times.

The heating of a dense quasineutral EHP was simulated numerically based on the quasi-hydrodynamic equations [1] in [2]. The situations in which the density and temperature distributions of the EHP under conditions of heating, though they turned out to be nonuniform, were characterized by scales of nonuniformity of the order of the dimensions of the system, were studied. However, heating of the EHP in systems whose dimensions are much larger than the diffusion length ℓ_0 and the cooling length of the charge carriers ℓ_ε can cause the system to become unstable with respect to fluctuations whose scale is comparable to ℓ_0 and ℓ_ε [3, 4]. This instability leads to stratification and separation of the EHP, i.e., to the appearance of dissipative structures in it.

This work is concerned with the numerical simulation of the dynamics of stratification of a dense quasineutral EHP in current-heated binary heterostructures. In our preceding work [5], we described the mathematical model used, and we also presented the algorithm for the numerical solution of this problem, so that here we shall briefly describe the mathematical formulation of the problem.

We are studying binary p–i–n heterostructures with wide-gap n and p regions and a narrow-gap i region, the charge carriers (electrons and holes) in which have the same effective masses. It is assumed that owing to the high density of electrons and holes injected into the i region, their energy distribution is Maxwellian with the same effective temperature Θ.

In the situation under study the EHP is characterized by the carrier density N and the effective carrier temperature Θ, which satisfy the following equations in the two-dimensional geometry:

$$\frac{\partial N}{\partial \tau} = \frac{\partial^2}{\partial X^2}[D(\Theta)N] + \frac{\partial^2}{\partial Z^2}[D(\Theta)N] + R(N, \Theta), \tag{1}$$

$$\frac{3}{2}\frac{\partial (N\Theta)}{\partial \tau} = \frac{\partial}{\partial X}\left\{\varkappa(N, \Theta)\frac{\partial \Theta}{\partial X} + \left(\frac{5}{2} + \alpha\right)\frac{\Theta\partial}{\partial X}[D(\Theta)N]\right\} + \frac{\partial}{\partial Z}\left\{\varkappa(N, \Theta)\frac{\partial \Theta}{\partial Z} + \left(\frac{5}{2} + \alpha\right)\frac{\Theta\partial}{\partial Z}[D(\Theta)N]\right\} + P(N, \Theta). \tag{2}$$

Here $\varkappa(N, \Theta) = (5/2 + \alpha)ND(\Theta)$, where α is a numerical coefficient determined by the energy dependence of the momentum relaxation time. The terms R(N, Θ) and P(N, Θ) describe the recombination of electrons and holes and their energy relaxation. Next we set

M. V. Keldysh Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 4, pp. 645-650, October, 1986. Original article submitted July 29, 1985.

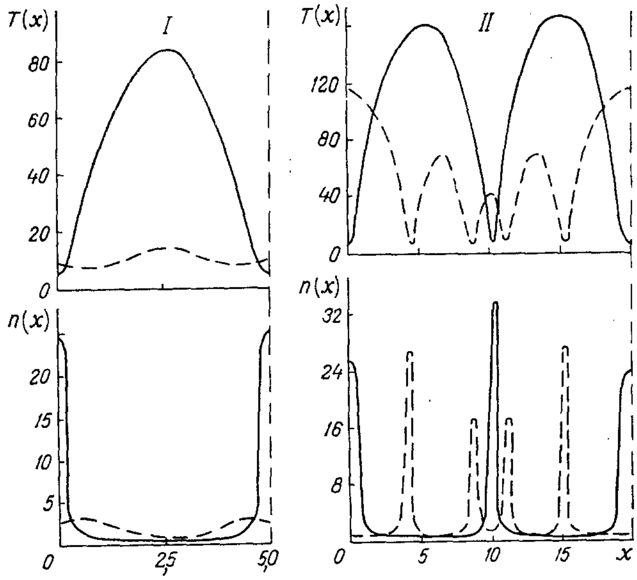

Fig. 1. Dynamics of the establishment of stationary temperature T and density n distributions: I) for ℓ = 5, the broken curves correspond to t = 0.01 and solid curves correspond to t ≥ 0.03; II) for ℓ = 20, the broken curves correspond to t = 0.06 and the solid curves correspond to t ≥ 0.6. All quantities are dimensionless.

$$R(N,\ \Theta)=\frac{N_l-N}{\tau_r(\Theta)},\ P(N,\ \Theta)=\frac{N(\Theta_l-\Theta)}{\tau_\varepsilon(\Theta)}. \tag{3}$$

Assuming that the active region is rectangular with dimensions ℓ_x and ℓ_z, the boundary conditions for Eqs. (1) and (2) are given in the form:

$$-\frac{\partial}{\partial Z}[D(\Theta)\,N]\Big|_{Z=0,l_z}=\pm J, \tag{4}$$

$$-\left\{\varkappa(N,\ \Theta)\frac{\partial\Theta}{\partial Z}+\left(\frac{5}{2}+\alpha\right)\frac{\Theta\partial}{\partial Z}[D(\Theta)N]\right\}\Big|_{Z=0,l_z}=\pm\Delta\cdot J, \tag{5}$$

$$\frac{\partial}{\partial X}[D(\Theta)\,N]\Big|_{X=0,l_x}=0, \tag{6}$$

$$\varkappa(N,\ \Theta)\frac{\partial\Theta}{\partial X}\Big|_{X=0,l_x}=0. \tag{7}$$

Here $J=J(t)\cdot X\left(\frac{a_x^2}{4}-\left(X-\frac{l_x}{2}\right)^2\right)$, and Δ is the magnitude of the jump of the bottom of the conduction band (and of the top of the valence band) in the heterojunctions. The initial uniform or perturbed density and temperature distributions are given.

In the case when the thickness of the active i region ℓ_z is much smaller than the diffusion length ℓ_D and the cooling length ℓ_ε, the following one-dimensional model, which makes it possible to study the density N and temperature Θ distributions in a direction perpendicular to the direction of current flow (i.e., in the plane of the heterostructure layers), is used:

$$\frac{\partial N}{\partial\tau}=\frac{\partial^2}{\partial X^2}[D(\Theta)\,N]+R(N,\ \Theta)+\frac{2J}{l_z}, \tag{8}$$

$$\frac{3}{2}\frac{\partial(N\Theta)}{\partial\tau}=\frac{\partial}{\partial X}\left\{\varkappa(N,\ \Theta)\frac{\partial\Theta}{\partial X}+\left(\frac{5}{2}+\alpha\right)\frac{\Theta\partial}{\partial X}[D(\Theta)\,N]\right\}+P(N,\ \Theta)+\frac{2J\Delta}{l_z}, \tag{9}$$

$$\frac{\partial}{\partial X}\left[D(\Theta)N\right]\Big|_{X=0,l_x}=0,\quad \varkappa(N,\Theta)\frac{\partial\Theta}{\partial X}\Big|_{X=0,l_x}=0, \tag{10}$$

$$N|_{\tau=0}=N_0+\delta N_0,\quad \Theta|_{\tau=0}=\Theta_0+\delta\Theta_0, \tag{11}$$

where N_0, Θ_0 are the uniform stationary solutions of Eqs. (8) and (9), while the quantities δN_0 and $\delta\Theta_0$ are the perturbations of the initial uniform states.

In the numerical calculations [5] the following dimensionless quantities were used:

$$\frac{N}{N_l}=n,\quad \frac{\Theta}{\Theta_l}=T,\quad \frac{X}{l_z}=x,\quad \frac{Z}{l_z}=z,\quad \frac{l_x}{l_z}=l,\quad \frac{a_x}{l_z}=a,$$
$$\frac{\tau D}{l_z^2}=t,\quad \frac{\tau_r D}{l_z^2}=\tau_R,\quad \frac{\tau_\varepsilon D}{l_z^2}=\tau_E, \tag{12}$$
$$\frac{J(\tau,X)l_z}{DN_l}=I(t,x),\quad \frac{\Delta}{\Theta_l}=\delta,$$

where $D = D(\Theta_\ell)$; $\tau_r = \tau_r(\Theta_\ell)$; $\tau_\varepsilon = \tau_\varepsilon(\Theta_\ell)$.

Using the one-dimensional model given by (8)-(11), we studied the dynamics of stratification of EHP in the active region. The following values of the parameters were chosen $\alpha = 1.5$; $r = 0$; $\varepsilon = -0.05$; $\tau_R = 10^{-2}$; $\tau_E = 10^{-4}$; $I = 50$. The initial conditions were given in the form

$$n(x,0)=n_0\left(1\pm 0.1\cos\frac{\pi kx}{l}\right),\quad T(x,0)=T_0\left(1\mp 0.1\cos\frac{\pi kx}{l}\right).$$

The calculations established that for $T_0 \leq T_1$, $\delta \leq \delta_1$ and $T_0 \geq T_2$, $\delta \geq \delta_2$ (for the chosen values of the parameters $T_1 \approx 3$, $T_2 \approx 14$), where T_0 and δ are related by the relation $T_0 - 1 = \delta\tau\varepsilon(T_0)/\tau_r$, i.e., for relatively weak and strong overheating of the EHP by the current, respectively, the given initial nonuniform perturbations (11) are smoothed out and spatially uniform stationary distributions of the EHP density and temperature are established. The values of T_1, δ_1 and T_2, δ_2 are essentially the values of the parameters for which bifurcation of the solution of the system (8)-(11) occurs.

For the case of relatively average overheating, in particular, when $T_0 = 10$, $n_0 = 2$, $\delta = 5600$, the establishment of stationary EHP density and temperature distributions was studied as a function of the length of the region ℓ. The calculations were performed for $\ell = 1$, 5, 10, 20, and 40. Figure 1 shows the typical stratified distributions (solid curves), characterized by nonuniformities which are much smaller than the length of the active region ℓ. It is evident from the figure that the number of strata (narrow cold regions with high density) increases as ℓ increases (for $\ell = 40$ there are five strata). It is interesting that the established nonuniform distributions of the effective temperature and concentration observed for $\ell < 20$ and leading to different stratified stationary distributions with the same parameters but slightly different perturbations in the initial conditions are not unique: $n(x, 0) = n_0(1 - 0.1 \cos 2\pi x/\ell)$, $T(x, 0) = T_0(1 + 0.1 \cos 2\pi x/\ell)$ and $n(x, 0) = n_0(1 + 0.1 \cos 2\pi x/\ell)$, $T(x, 0 = T_0(1 - 0.1 \cos 2\pi x/\ell)$. As the length of the structure increases ($\ell \geq 20$) for different initial conditions the same stationary EHP density and effective temperature distributions are established, though these stratified distributions evolve differently. The times for establishing stationary distributions (in units of the characteristic energy relaxation time of the carriers) are as follows:

l	1	5	10	20	40
t/τ_E	500	1000	1250	5000	5000

If the size (thickness) of the active region in the direction of current flow ℓ_z is not small compared with the diffusion length ℓ_D and the cooling length ℓ_ε, then the EHP density and effective temperature distributions have a two-dimensional character. For relatively small overheatings of the EHP stationary distributions whose scale of nonuniformity is comparable to the geometric dimensions of the active region of the heterostructure are established. Figure 2 shows the contour lines of constant temperature of the EHP in the active

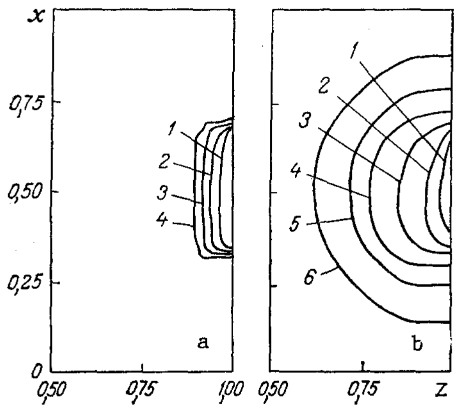

Fig. 2

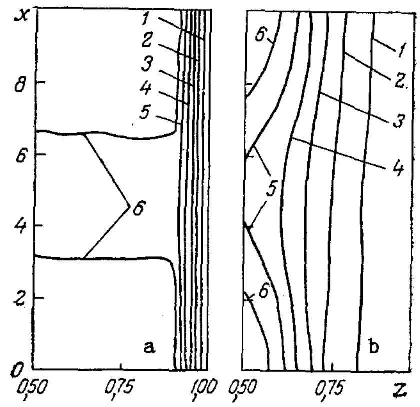

Fig. 3

Fig. 2. Example of results of calculations for the case of weak overheating of the EHP. The lines of constant temperature are given for the following times: a) $t = 5 \cdot 10^{-4}$, 1) T = 8.827; 2) 8.952; 3) 8.976; 4) 8.993; b) $t = 6 \cdot 10^{-2}$, 1) T = 8.858; 2) 8.895; 3) 8.949; 4) 8.986; 5) 9.004; 6) 9.023. All quantities are dimensionless.

Fig. 3. Example of calculations for the case of quite strong overheating of the EHP. The lines of constant density at different times: a) $t = 9 \cdot 10^{-7}$, 1) n = 0.458; 2) 0.749; 3) 1.039; 4) 1.329; 5) 1.619; 6) 1.909; b) $t = 4.42 \cdot 10^{-4}$, 1) n = 0.991; 2) 1.573; 3) 2.157; 4) 2.740; 5) 3.324; 6) 3.907. All quantities are dimensionless.

region (in view of the symmetry of the problem only half the region is shown in the figure) at different times for the following values of the parameters: $\ell = 1$, $a = 1/3$, $I = 50$, $\delta = 12.1$; $\tau_R = 4 \cdot 10^{-3}$; $\tau_E = 1.12 \cdot 10^{-1}$; $n_0 = 2.8$; $T_0 = 9$. The distributions for $t \geq 0.06$ already correspond to the steady state.

In the case of quite strong overheating of the EHP a tendency toward stratification of the density and temperature distributions, i.e., toward the formation of nonuniformities of relatively small scale, is clearly observed like in the one-dimensional case. The lines of constant EHP density for this case ($\ell = 10$, $a = 10$, $\tau_R = 10^{-2}$, $\tau_E = 10^{-4}$, $I = 50$, $n_0 = 2(1 - 0.1 \cos 2\pi x/\ell)$, $T_0 = 10(1 + 0.1 + 0.1 \cos 2\pi x/\ell))$ at different times are shown in Fig.3. Unfortunately, it is impossible to follow the establishment of stationary stratified distributions in a two-dimensional geometry because of the excessive amount of computer time required (for this variant the maximum achievable time step in the calculations is only 10^{-8}).

Thus, with the help of a computational experiment it was shown that depending on the magnitude of the pumping current and therefore the overheating of the EHP both uniform and stratified stationary density and effective temperature distributions are possible. In the spatially one-dimensional case the form of the stratified distributions as a function of the length of the active region of the heterostructure was established.

NOTATION

ℓ_D, diffusion length; ℓ_ε, cooling length of the carriers; N and n, carrier (electron and hole) densities; Θ, T, are the effective temperatures; $D(\Theta)$, $\varkappa(N, \Theta)$, coefficients of diffusion and thermal conductivity of the EHP; $R(N, \Theta)$, $P(N, \Theta)$, recombination and relaxation terms; $\tau_r(\Theta)$, $\tau_\varepsilon(\Theta)$, characteristic lifetimes and energy relaxation times of the carriers; N_ℓ density of EHP in the absence of injection through the heterojunctions; Θ_ℓ, lattice temperature; ℓ_x, ℓ, ℓ_z, dimensions of the active region; a_x and a, dimensions of the injecting contact; X, x, Z, and z, spatial coordinates; J and I, densities of the injected current; Δ, δ, energies of the injected carriers; $X(x)$, Heaviside function; and: τ and t, times.

LITERATURE CITED

1. F. G. Bass and Yu. G. Gurevich, Hot Electrons and Strong Electromagnetic Waves in Semiconductor and Gas-Discharge Plasmas [in Russian], Moscow (1975).

2. V. A. Nikolaeva, V. I. Ryzhii, and B. N. Chetverushkin, "Numerical modeling of a dense electron—hole plasma in binary heterostructures," Preprint No. 38, IPM, Akad. Nauk. SSSR, Moscow (1985).
3. B. S. Kerner and V. V. Osipov, Zh. Eksp. Teor. Fiz., 71, 1542-1554 (1976).
4. B. S. Kerner and V. V. Osipov, Fiz. Tekh. Poluprovodn., 13, 721-734 (1979).
5. B. A. Nikolaeva, V. I. Ryzhii, and B. I. Chetverushkin, Inzh.-Fiz. Zh., 51, No. 3, 492-501 (1986).

STABILITY AND FORM OF AN ARC COLUMN IN A TRANSVERSE GAS FLOW

S. G. Lisitsyn and N. V. Pashatskii

UDC 621.3.014.31

The analytic conditions for stable burning of an electric arc in a gas flow are obtained. The form of the arc column in a transverse gas flow is determined.

A great deal of attention is being devoted to the problem of the stability of an electric arc. A number of authors [1-3] have studied the spatial stability of an arc neglecting the effect of the electric circuit by introducing the concepts of mass and nonmass forces. Others have studied the energy stability of electric arc circuits without a gas flow [4-6] and with a gas flow [7-8], neglecting the details of the motion of the arc in space. The spatial and energy stability of different electric arcs is studied simultaneously in [9] using the "force" model. The results on the stability are based on an analysis of the forces which act on the arc and which are difficult to determine in experiments.

The form of the arc column has been studied quite completely for an arc in a longitudinal flow. As regards an arc in a transverse flow, the form of the arc is determined primarily experimentally.

We shall study a cylindrical arc stabilized by electrodes in a transverse gas flow. We shall assume that there is no external magnetic field, while the intrinsic magnetic field is negligibly small. Then the equation expressing the balance of energy in the arc for a linear dependence of the gas properties on the enthalpy has the form [10]

$$\frac{\partial \overline{h}}{\partial t}+(\mathbf{w},\ \nabla \overline{h})=a^2\nabla^2\overline{h}+(\varepsilon^2-\overline{\varepsilon^2})\,\overline{h}, \tag{1}$$

where $\overline{\varepsilon^2}$ takes into account the radiative heat losses. We assume that the intensity of the electric field in the arc ε depends on t and varies only along the arc, since, as investigations have shown [5, 6], the arc is stable with respect to the transverse perturbations of the field.

Averaging Eq. (1) over the period of oscillations of the current in the circuit, if the arc is an ac current arc, and integrating over the cross section of the arc column, in the presence of perturbations in the system we obtain

$$\frac{\partial H}{\partial t}=a^2\,\frac{\partial^2 H}{\partial x^2}+p-q+F(x,\ t), \tag{2}$$

where p and q are, respectively, the Joule heat and heat losses to the environment per unit arc length; F(x, t) is the external perturbation. The overbar indicating averaging over the period is dropped here and below.

Equation (2) must be supplemented by Ohm's law for the circuit and relations for p and q:

$$\int\limits_{-l/2}^{l/2}\varepsilon(x,\ t)\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2}\,dx=l\left(\varepsilon_s-\frac{RI}{l}\sqrt{\frac{\overline{\sigma}}{\overline{M}}}\right), \tag{3}$$

Moscow Engineering Physics Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 4, pp. 650-654, October, 1986. Original article submitted July 22, 1985.

 © 1987 Plenum Publishing Corporation